Recall: Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$	$\Rightarrow \sin^2 x = 1 - \cos^2 x$ * $\Rightarrow \cos^2 x = 1 - \sin^2 x$
$\sec^2 x = 1 + \tan^2 x$	$\Rightarrow \tan^2 x = \sec^2 x - 1$

rule: use when given ODD powers of sine or cosine

Double Angle (Reduction) Formulas

$\sin 2x = 2\sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$	$\Rightarrow \cos^2 x = \frac{1}{2}(\cos 2x + 1)$
$\cos 2x = 1 - 2\sin^2 x$	$\Rightarrow \sin^2 x = \frac{1}{2}(1 - \cos 2x)$

rule: use when given EVEN powers of sine or cosine

ODD POWER

Do: $\int \sin^3 x\, dx$ $= \int \sin^2 x \underbrace{\sin x\, dx}_{du}$ *

$u = \cos x$

$-du = \sin x\, dx$

$(\cos x)' = -\sin x$

$= \int (1 - \cos^2 x) \sin x\, dx$

$= -\int (1 - u^2)\, du$

$= -\left(u - \frac{u^3}{3}\right) + C$

$= \boxed{-\cos x + \frac{1}{3}\cos^3 x + C}$

recall: $\int \frac{dx}{x^2\sqrt{1-x^2}}$ write ITO θ

$x = \sin\theta$ $dx = \cos\theta\, d\theta$

$x^2 = \sin^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \cos\theta$

$= \int \frac{\cos\theta\, d\theta}{\sin^2\theta\sqrt{1-\sin^2\theta}}$

$= \int \frac{\cancel{\cos\theta}}{\sin^2\theta\,\cancel{\cos\theta}}\, d\theta$

reciprocals

$\sin\theta \Leftrightarrow \csc\theta \quad \therefore \frac{1}{\sin\theta} = \csc\theta$

$= \int \frac{1}{\sin^2\theta}\, d\theta$

$-(\cot\theta)' = +\csc^2\theta$

$= \int \csc^2\theta\, d\theta$

$= -\cot\theta + C$ re-write ITO x

$x = \sin\theta \Rightarrow \sin\theta = \frac{x}{1}\ \frac{O}{H}$

$= -\frac{\cos\theta}{\sin\theta} + C$

$\therefore \cos\theta = \frac{A}{H} = \sqrt{1-x^2}$

$= -\frac{\cos\theta}{x} + C = \boxed{-\frac{\sqrt{1-x^2}}{x} + C}$

$x^2 + A^2 = 1^2$

ex. $\int \frac{dx}{x^2\sqrt{4-x^2}}$

$x = 2\sin\theta \longrightarrow dx = 2\cos\theta\, d\theta$

$x^2 = (2\sin\theta)^2$

$= 4\sin^2\theta$

$= \frac{2}{4}\int \frac{\cos\theta}{\sin^2\theta\sqrt{4-4\sin^2\theta}}\, d\theta$

$\sqrt{4-4\sin^2\theta}$

$= \sqrt{4(1-\sin^2\theta)}$

$= \sqrt{4}\sqrt{1-\sin^2\theta}$

$= 2\sqrt{1-\sin^2\theta}$

$= 2\cos\theta$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= \frac{1}{2}\cdot\frac{1}{2}\int \frac{\cancel{\cos\theta}}{\sin^2\theta\,\cancel{\cos\theta}}\, d\theta$

$= \frac{1}{4}\int \frac{1}{\sin^2\theta}\, d\theta$

Do: INTEGRATE

(WAIT ON SUBSTITUTION BACK TO X)

$= \frac{1}{4}\int \csc^2\theta\, d\theta$

$= -\frac{1}{4}\cot\theta + C$

$= -\frac{1}{4}\frac{\cos\theta}{\sin\theta} + C$

$x = 2\sin\theta \Rightarrow \frac{x}{2} = \sin\theta$

$\sqrt{4-x^2} = 2\cos\theta \Rightarrow \frac{\sqrt{4-x^2}}{2} = \cos\theta$

$= -\frac{1}{4}\cdot\frac{\frac{\sqrt{4-x^2}}{\cancel{2}}}{\frac{x}{\cancel{2}}} + C$

KCF

$= \boxed{-\frac{\sqrt{4-x^2}}{4x} + C}$

from pg 1: $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

EVEN

recall: $\int \sin^2 x \, dx = \frac{1}{2}\int (1 - \cos 2x)\, dx$

$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$

OR

$= \boxed{\frac{1}{2}x - \frac{1}{4}\sin 2x + C}$

~~ex. $\int \sin^4 x \cos^4 x \, dx$~~

recall: $\int \tan x\, dx = \int \frac{\sin x}{\cos x}\, dx$

$= \int \frac{1}{\underbrace{\cos x}_{u}} \cdot \underbrace{\sin x\, dx}_{du}$

$= -\int \frac{1}{u}\, du$

$= -\ln|u| + C$

$= \boxed{-\ln|\cos x| + C}$

$u = \cos x$

$-du = \sin x\, dx$

$(\ln u)' = \frac{1}{u}$

ex. $\int \sec x\, dx = \int \sec x \left(\frac{\sec x + \tan x}{\sec x + \tan x} \right) dx$

"1"

distribute

$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$

$= \int \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}\, dx$

$u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x\, dx$

$= \int \frac{1}{u}\, du$

$= \ln|u| + C = \boxed{\ln|\sec x + \tan x| + C}$

From P.1

$\sec^2 x = 1 + \tan^2 x$

$\therefore \tan^2 x = \sec^2 x - 1$

ex. $\int \tan^2 x\, dx = \int (\sec^2 x - 1)\, dx$

$= \int \sec^2 x\, dx - \int 1\, dx$

$= \boxed{\tan x - x + C}$

$(\tan x)' = \sec^2 x$

ex. $\int \tan^3 x\, dx = \int \underbrace{\tan^2 x}\, \tan x\, dx$ distribute

$= \int (\sec^2 x - 1)\tan x\, dx$

$= \int (\sec^2 x \tan x - \tan x)\, dx$

$u = \tan x$
$du = \sec^2 x\, dx$

$= \int \tan x \sec^2 x\, dx - \int \tan x\, dx$

$= \int u\, du - \int \frac{\sin x}{\cos x}\, dx$ $\quad u = \cos x$, $-du = \sin x\, dx$

$= \frac{u^2}{2} + \int \frac{1}{u}\, du$

$= \boxed{\frac{1}{2}\tan^2 x + \ln|\cos x| + C}$

ex. $\int \sec^3 x\, dx = \int \underbrace{\sec x}_{u}\, \underbrace{\sec^2 x\, dx}_{dv}$ use IBP !!

$u = \sec x$, $du = \sec x \tan x\, dx$ $\quad dv = \sec^2 x\, dx$, $v = \tan x$

$\int u\, dv = uv - \int v\, du$

$= \sec x \tan x - \int \tan x \cdot \sec x \tan x\, dx$

$= \sec x \tan x - \int \sec x \cdot \underbrace{\tan^2 x}\, dx$

$= \sec x \tan x - \int \sec x (\sec^2 x - 1)\, dx$ DISTRIBUTE

$= \sec x \tan x - \int (\sec^3 x - \sec x)\, dx$

$\int \sec^3 x\, dx = \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$ combine like terms

$+\int \sec^3 x\, dx \qquad\qquad +\int \sec^3 x\, dx$

$2\int \sec^3 x\, dx = \sec x \tan x + \int \sec x\, dx$

$\frac{\cancel{2}\int \sec^3 x\, dx}{\cancel{2}} = \frac{\sec x \tan x + \ln|\sec x + \tan x| + C}{2}$

constant "absorbs" $\frac{1}{2}$

$\int \sec^3 x\, dx = \boxed{\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C}$